

Quantum origin of suppression for vacuum fluctuations of energy

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By example of a model with a spatially global scalar field, we show that the energy density of zero-point modes is exponentially suppressed by an average number of field quanta in a finite volume with respect to the energy density in the stationary state of minimal energy. We describe cosmological implications of mechanism.

I. INTRODUCTION AND RECAPITULATION OF COSMOLOGICAL CONSTANT PROBLEM

After primary speculations presented in [1], the cosmological constant [2] is associated with a vacuum energy density generated by zero-point modes of quantum fields [3, 4]. In the framework of quantum field theory, these quantum fluctuations are ordinary divergent and they should be renormalized. In this respect, the relevant renormalization is related with actual thresholds of energies, at which particles and forces contribute significantly. Then, usually one supposes that some combinations of Planckian scale and particle masses generate the energy density of vacuum ρ_{vac} , so that the maximal estimate corresponds to the greater scale known in the real physics and it yields $\rho_{\text{vac}} \sim m_{\text{Pl}}^4$, where the reduced Planck mass $m_{\text{Pl}} \approx 2.4 \cdot 10^{18}$ GeV is given by the Newton constant G as $8\pi G m_{\text{Pl}}^2 = 1$. Such kind of estimates is in the direct conflict with the value extracted from the cosmological data [5] giving $\rho_{\text{vac}} \mapsto \rho_{\Lambda} = \Lambda^4$ at $\Lambda \sim 10^{-3}$ eV, that is 30 orders of magnitude less than the Planck mass.

However, nobody can guarantee the the Planck scale defining the strength of gravitational interaction has to establish a fundamental mass scale or energy threshold relevant to the cosmological constant, of course. In this respect, one could follow more realistic way by using the well accomplished description of particle physics in the Standard Model. So, the direct observation of Higgs boson allows us to evaluate the energy density of electroweak vacuum from the effective potential of Higgs boson in terms of masses of Higgs boson and W boson, that gives $\rho_{\Lambda}^{\text{EW}} \sim m_H^2 m_W^2 \sim (10^2 \text{ GeV})^4$, which exceeds the contribution due to the additional condensates in the Quantum Chromodynamics by 8 orders of magnitude, at least. Then, the magnitude of mismatching the scale of cosmological constant would be significantly relaxed from 30 orders to 14 orders, that is still a big deal, no doubt. Notice, that in this approach to the estimate of cosmological constant scale, one ignores an arbitrary constant shift of Higgs potential. This shift can originate from physics of other fields. In addition, at the observed mass value the Higgs potential can lose its stability at very large fields below the Planckian range due to effects of renormalization group, that could produce the tunnel decay of present Universe to another Universe with a different vacuum.

Let us show that such the suppression can be explained due to a finite volume effect for quantum

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fluctuations in an excited non-stationary state. For the sake of clarity we exhibit the mechanism by considering a time-dependent scalar field $\phi(t)$, which is spatially global and free. The action in a finite physical volume V_R is given by expression

$$S = V_R \int dt \frac{1}{2} (\dot{\phi}^2 - m^2 \phi^2), \quad (1)$$

where $\dot{\phi} = d\phi/dt$ and m is the field mass. The specific reference frame of space-time suggestively should be associated with the reference frame of homogeneous component of cosmic microwave background radiation in the Universe, so that time t could correspond to the cosmic time of Friedmann–Robertson–Walker–Lemaitre metrics. The meaning of V_R is the volume wherein the fluctuations of field are causal, so that it can be considered as spatially global, while an inhomogeneity is evaluated by $|\nabla\phi| \sim \delta\phi/\lambda_c$, where λ_c is the Compton length, $\lambda_c = 1/m$, and $\delta\phi$ denotes the field fluctuation. The basic motivation and consideration are further considered in the field model of (1), so we ignore the influence of curved space-time on the main effect for the moment. However, we return to the discussion of this issue in Section III.

Formally, action (1) corresponds to the harmonic oscillator of “frequency” m and “inertial mass” V_R . Then, the dimensionless operators

$$\hat{Q} = \frac{\phi}{\phi_0}, \quad \hat{P} = \frac{\dot{\phi}}{\dot{\phi}_0},$$

at

$$\phi_0^2 = \frac{1}{V_R m}, \quad \dot{\phi}_0^2 = \frac{m}{V_R},$$

define the operators of annihilation and creation for the spatially global field quanta

$$\hat{a} = \frac{1}{\sqrt{2}}(\hat{Q} + i\hat{P}), \quad \hat{a}^\dagger = \frac{1}{\sqrt{2}}(\hat{Q} - i\hat{P}),$$

with the standard commutator

$$[\hat{a}, \hat{a}^\dagger] = 1.$$

The hamiltonian takes the form

$$\hat{H} = \frac{1}{2} V_R (\dot{\phi}^2 + m^2 \phi^2) = \frac{1}{2} m (\hat{P}^2 + \hat{Q}^2) = m \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right). \quad (2)$$

If the field is non-stationary excited, its quantum state can be considered as a superposition of oscillatory coherent states, which minimize uncertainties in the field ϕ and its rate $\dot{\phi}$. Let us consider the coherent state $|\alpha\rangle$ with the average number of quanta n in the volume V_R :

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle, \quad \alpha^* \alpha = n, \quad \alpha = \frac{1}{\sqrt{2}}(Q_0 + iP_0).$$

The averaged energy reads off

$$\langle E \rangle = \langle \alpha | \hat{H} | \alpha \rangle = m \left(n + \frac{1}{2} \right).$$

Usually, the minimal energy shift of this state from the minimum of potential is referred to the energy level of zero-point mode,

$$\delta_{\min} E = \frac{1}{2} m. \quad (3)$$

So, it defines the quantity, which we call the bare cosmological constant,

$$\rho_{\Lambda}^{\text{bare}} = \frac{m}{2V_R} = \langle \text{vac} | \rho | \text{vac} \rangle = m^2 \langle \text{vac} | \phi^2 | \text{vac} \rangle = \langle \text{vac} | \dot{\phi}^2 | \text{vac} \rangle. \quad (4)$$

This shows that the finite volume sets non-zero fluctuations of spatially global field.

If the fluctuations of field match to the Planckian scale, $\langle \text{vac} | \phi^2 | \text{vac} \rangle \sim m_{\text{Pl}}^2$, then the bare cosmological constant $\rho_{\Lambda}^{\text{bare}}$ takes a huge value, that constitutes the cosmological constant problem. On the other hand, if we restrict ourselves by the range of Standard Model, then we would expect that the mass of scalar field is given by the mass of Higgs boson M , while the fluctuations of the field square are of the order of natural scale in the electroweak physics, i.e. M^2 . So, one could arrive to the estimate that is consistent with the expectations of Standard Model, but it still would get a huge value.

However, in the next section we show that if the number of quanta for $\phi(t)$ is not equal to zero, $n \neq 0$, then only a fraction of energy shift from the potential minimum refers to the zero-point mode and the fractional part of energy does originate from the suppressed vacuum fluctuations, hence, the suppressed fraction corresponds to the vacuum energy density, indeed.

II. SUPPRESSION MECHANISM IN ACTION

Let us find the fraction of zero-point mode in the energy shift from the minimum of potential for the coherent state exactly. So, decomposing the state into the sum of vacuum $|\text{vac}\rangle$ and the state $|\text{quanta}\rangle$ with nonzero numbers of stationary field quanta

$$|\alpha\rangle = \mathcal{A}_{\text{vac}} |\text{vac}\rangle + \mathcal{A}_q |\text{quanta}\rangle,$$

we evaluate the average density of energy

$$\langle \alpha | \rho | \alpha \rangle = |\mathcal{A}_{\text{vac}}|^2 \langle \text{vac} | \rho | \text{vac} \rangle + |\mathcal{A}_q|^2 \langle \text{quanta} | \rho | \text{quanta} \rangle, \quad (5)$$

where the probability to find k quanta in the coherent state is given by the Poisson distribution,

$$|\mathcal{A}_k|^2 = \frac{n^k}{k!} e^{-n},$$

while the free hamiltonian does not mix the stationary states with different numbers of quanta, of course.

Therefore, the average density of energy, which is observed in the gravity, is decomposed as

$$\langle \rho \rangle = |\mathcal{A}_{\text{vac}}|^2 \rho_{\Lambda}^{\text{bare}} + \rho_q, \quad (6)$$

at

$$|\mathcal{A}_{\text{vac}}|^2 = e^{-n}, \quad (7)$$

and ρ_q being the energy density of nonzero-point modes that for the coherent state equals

$$\rho_q = \frac{m}{V_R} \left(n + \frac{1}{2} - \frac{1}{2} e^{-n} \right) = \rho_\Lambda^{\text{bare}} (2n + 1 - e^{-n}).$$

Thus, the true energy density generated by zero-point modes in the coherent state is suppressed and it is given by

$$\rho_\Lambda = |\mathcal{A}_{\text{vac}}|^2 \rho_\Lambda^{\text{bare}} \mapsto e^{-n} \rho_\Lambda^{\text{bare}}. \quad (8)$$

Relation (8) remains valid generically by the order of magnitude not only for the coherent state, but also for the most ordinary, non-exotic states yielding $|\mathcal{A}_{\text{vac}}|^2 \sim e^{-n}$. Anyway, we can hold the relation for the probability of zero-point mode in the quantum state as the definition of effective number of quanta in this state. Moreover, if the fluctuations of field quanta are *statistically occasional*, then the probability with respect to the number of quanta has to fit the *Poisson distribution*, and hence, the quantum state should be the coherent state.

Let us stress that the described effect of vacuum fluctuations suppression in the excited non-stationary state necessary involves the finite volume, but it has no connection to the well known Casimir effect, which is also related with a restricted volume. Indeed, the Casimir effect takes place due to the relevant modification of zero-point modes in the state of *minimal* energy for the system of restricted volume, while setting the state of minimal energy in our consideration will mean $n \rightarrow 0$ and the suppression factor will disappear, that results in the ordinary situation for the Casimir effect. In other words, the suppression factor becomes essential if the system is excited to the non-stationary state, when the Casimir effect is irrelevant, and vice versa, the Casimir effect takes place, when the suppression under study is not in action. In this respect, the problem of cosmological constant comes back in force, when we deal with the stationary state of minimal energy, while near this state the Casimir effect represents the evidence for the reality of vacuum energy.

The meaning of bare cosmological constant is the following: if the system would be in the very vacuum state, the bare cosmological constant would be the energy density in the system. i.e. in the empty vacuum without any fields, particles and quanta, just vacuum fluctuations only. If the system is not empty and it is occasionally excited, the actual vacuum fluctuations are suppressed, that mean the suppression of observed cosmological constant in such the word. In this way, we assume in our model that there is no any different contribution to the cosmological constant, say, like some induced terms of various nature, since those additional terms cannot be suppressed in the same manner.

The decomposition of (6)–(8) would remain formal, if the field is held free and it does not interact, and then $\rho_\Lambda^{\text{bare}}$ would set the minimal density of energy, of course, while we would observe the total density of field energy without a possibility to extract the energy density of suppressed zero-point fluctuations. However, if the field interacts, then it goes a non-trivial evolution: the quanta can mix and transform into quanta of matter fields, while the vacuum transfers itself into itself, i.e. into the *vacuum*, since it is *stable*, hence, the contribution of zero-point modes into the energy density remains constant with the evolution, and this contribution is much less than the bare term because of excited state of field, that makes decomposition in (6)–(8) to be observable for the interacting field¹. The suppressed contribution of zero-point modes is observed as the

¹ The energy-momentum tensor with interactions acts as the source of matter production, hence, it can cause the creation of quanta from the vacuum. However, this effect cannot influence on the contribution of zero-point modes themselves into its energy density, surely.

cosmological constant in the presence of matter quanta.

In other words, the decomposition of (6)–(8) is based on quantum mechanics, and it is detectable. The detection suggests the interaction of field system. Roughly speaking, the field quanta decay to visible particles of matter, while the suppressed vacuum fluctuations form the observable cosmological constant.

Let us look at the pressure of zero-point modes in order to justify their vacuum status. So, the bare zero-point modes have got the energy $E^{\text{bare}} = \frac{1}{2}m$ independent on the reference volume. It means that the pressure is given by $p^{\text{bare}} = \partial E^{\text{bare}} / \partial V_R \equiv 0$, that can be also calculated by means of averaging the spatial components of energy-momentum tensor,

$$\langle \text{vac} | T_{\alpha}^{\beta} | \text{vac} \rangle = -\delta_{\alpha}^{\beta} p^{\text{bare}} = -\delta_{\alpha}^{\beta} \frac{1}{2} \langle \text{vac} | \left\{ \dot{\phi}^2(t) - m^2 \phi(t) \right\} | \text{vac} \rangle = 0.$$

In contrast, the true energy of suppressed vacuum fluctuations in volume V_R

$$E_{\text{vac}} = V_R \rho_{\Lambda} = \frac{1}{2} m |\mathcal{A}_{\text{vac}}|^2$$

can get the correct dependence on the volume, if we put the vacuum density of energy to be constant, that implies $|\mathcal{A}_{\text{vac}}|^2 / V_R = \text{const.}$ and the pressure gets the value as it should be in the vacuum,

$$p = -\frac{\partial E_{\text{vac}}}{\partial V_R} = -\rho_{\Lambda},$$

hence,

$$\frac{\partial n}{\partial \ln V_R} = -1.$$

Therefore, the reference volume exponentially declines with the growth of quantum number n , and there is a maximal number corresponding to a minimal volume of Planckian length.

This derivation of actual value for the parameter of vacuum state is elementary, but it could be absolutely impossible, if we would ignore the variation of suppression factor in contrast to the case of stationary ground state.

Let us evaluate the relative inhomogeneity of field with respect to the energy density. So,

$$\frac{|\nabla \phi|^2}{m^2 \langle \phi \rangle^2} \sim \frac{m^2 (\delta \phi)^2}{m^2 \langle \phi \rangle^2} \sim \frac{\delta E}{E} \sim \frac{1}{\sqrt{n}} \ll 1.$$

Therefore, the inhomogeneity is negligible, if the field is non-stationary excited to a large value of quanta.

III. COSMOLOGY AND MODEL ESTIMATES

As we have already emphasized we have ignored effects due to a curved space-time, while we have derived our mechanism for the suppressed cosmological constant despite it is relevant to the system with gravity, indeed. In this respect, we assume that the relevant quantities can enter as the *initial conditions* for the further evolution of system by taking into account the gravitational expansion. So, the energy density of vacuum remains constant, while the quanta and its energy densities follow the transformations in accordance with relative field equations taking into account for the gravity, too.

Nevertheless, we have to mention that in the literature there are computations of energy density for the zero-point modes (ZPM) in a curved background, that take into account the dependence on the space-time curvature (see, for instance, the textbook by Birrell and Davies [6]). So, modern investigations in [7, 8] argue for the fact that in the curved space-time, for instance, in the de Sitter space-time being close to the space-time of inflation in the early Universe the zero-point modes themselves produce the energy density that quadratically evolves with the Hubble rate. We stress that such the effect means that the energy density of ZPM is not the cosmological constant at all, since the emergent equation of state (EOS) deviates from the vacuum equation of state, when the ratio of pressure to the energy density equals -1 , and, hence, zero-point modes generate the form of dark energy. So, one gets the opportunity to evaluate the relevant parameter of EOS for such the dark energy. The effect found in [7, 8] essentially changes the energy density of ZPM if the Hubble rate H exceeds the scale of ZPM energy density in the limit of $H \rightarrow 0$, i.e. in the flat space-time. In this respect we expect that this influence of space-time evolution on the ZPM energy density could be suppressed as the energy density divided by second degree of Planck mass and second degree of huge energy scale in the bare cosmological constant. In addition, the effect found in [7, 8] corresponds to the local fields considered in the whole space-time, including Hubble and super-Hubble distances, when, for instance, the dynamics of light scalar fields would be essential [8]. In contrast, in our model we deal with the global field in the limited volume, which is much less than the Hubble volume, when the approximation of field with the ordinary machinery of particle representation is very close to the exact solutions at such the distances deeply inside the Hubble horizon. So, we hope that the dynamical aspects of ZPM energy density would not be crucial for the scheme offered in the present paper.

In this context, there are similar and more general arguments in favor of situation, when the vacuum energy cannot be the constant value and, hence, it would never represent the cosmological constant, since the energy of vacuum in the curved space-time evolves due to the renormalization group equations with the Hubble rate as the evolution parameter [9–13]. Making use of conformal anomaly and other constructions in models, such the investigations [14, 15] argue for the dynamical vacuum energy, that at present can be tested by precision data in cosmology. Again, these studies deal with the dark energy, but not the cosmological constant, which is considered in our paper.

In our treatment of suppression factor, the calculations concerning for zero-point modes in the curved space-time would change an exact value of bare cosmological constant, however, the mechanism itself starts to work if the system is essentially excited to the occasional non-stationary state, that yields the suppression factor of bare cosmological constant even in the presence of space-time curvature, of course. So, in our study we hold the standard point of view on the vacuum energy equivalent to the cosmological constant and do not involve the dynamical treatment of vacuum energy evolving with the Universe expansion. In principle, we see that the offered mechanism can be implemented in the studies with the dynamical vacuum energy, too, simply by the insertion of suppression factor for the evolving value of vacuum energy, that could be considered in further developments of the presented way, elsewhere.

Further, another aspect of curved space-time is the particle creation, say, during the cosmological evolution [6]. Such the creation changes the energy density of matter by an additional term depending on the square of primary density of particles or Hubble rate in the fourth degree, that is typical for the quantum effects in the curved space-time. In this respect, the additional term is suppressed as the primary density of energy to the Planck mass in the fourth degree. It means that such the contribution is negligible if the energy density is significantly below the Planckian density, that is assumed for the Universe beyond the region of quantum gravity, of course. Moreover, such

the gravitational creation of matter does not influence the initial cosmological constant established at the start of evolution. Thus, we expect that our model can be considered in the cosmological aspects.

In the form of expression (8) the described mechanism rigorously sets the quantum suppression of bare cosmological constant. It is relevant to the cosmology because, at first, the spatially global scalar field could be associated with the spatially global part of inflaton [16–19], second, a finite volume of causal fluctuations corresponds to a primary volume of Universe at the inflation start, wherein the inflaton field can be considered as a spacially global². In this way, we can make estimations in a simple manner, say, by setting the primary fluctuations of field as

$$\langle \text{vac} | \phi^2 | \text{vac} \rangle \sim m_{\text{Pl}}^2 \quad \Rightarrow \quad \frac{1}{V_R} \sim m m_{\text{Pl}}^2,$$

hence, the average number of spatially global field quanta is evaluated by

$$n = \ln \frac{\rho_{\Lambda}^{\text{bare}}}{\rho_{\Lambda}} \sim 275 - 2 \ln \frac{m_{\text{Pl}}}{m} \gg 1,$$

since the inflaton is quite heavy, i.e. $m \sim 10^{13}$ GeV, and $n \sim 250$.

Because the mechanism should be accepted as actual and justified, the true question appears: What is the magic number of n ? Our studies show that the answer can be found, for instance, in the framework of model with the inflaton non-minimally coupled to the gravity, i.e. due to the interaction term of lagrangian in the form

$$\mathcal{L}_{\text{int}} = \frac{1}{2} \xi \phi m_{\text{Pl}} R,$$

where R is the scalar curvature of metric in the Jordan frame [20–24]. In the Einstein frame, the inflation scale is $\Lambda_{\text{inf}} \sim 10^{16}$ GeV, while the transformed inflaton gets the mass of the order of Λ_{inf}/ξ . The parameters obey the relation for the strong coupling³

$$n \sim \xi \sim \frac{m_{\text{Pl}}}{\Lambda_{\text{inf}}}.$$

Note that $\xi \gg 1$ results in a strong suppression of amplitude in a spectrum of relict gravitational waves, if the inflaton potential is exactly quadratic, when it satisfies the form of cosmic attractor for parameters of inflation [22, 24]. A valuable amplitude of relict gravitational waves would point to that the potential should involve some non-quadratic terms breaking the attractor predictions at $\xi \gg 1$. This amplitude of primary gravitational waves could be unambiguously extracted from the detection of B-modes of cosmic microwave background radiation, if the foreground polarization generated by the dust is suppressed in a region of detection. At present, the enforced amplitude of B-mode of cosmic microwave background radiation detected by BICEP2 [25] corresponds to such the secondary foreground produced by the measured dust distribution as the Planck collaboration has reported in [26].

Thus, the issue on the enigmatic value of n is transformed into a reasoning to fix the hierarchy of $\Lambda_{\text{inf}} \ll m_{\text{Pl}}$, in fact. Why does the inflation involve two energy scales? An answer would solve the cosmological constant problem [9, 27–29] by product, to our opinion.

² The volume of causal fluctuations is not equivalent to the Hubble volume determined by the initial density of energy. The Hubble volume could be greater than the finite volume of causal fluctuations, of course. Therefore, the inflaton can get a valuable inhomogeneity in the initial Hubble volume.

³ We are going to present the deeper consideration elsewhere.

IV. DISCUSSION AND GENERALIZATIONS

Let us argue for the relevance of spatially global scalar field to the cosmological constant. In the framework of quantum field theory the sum of divergent contributions of any existing fields into the vacuum energy density should be treated as a new independent global dimensional quantity with zero charges of vacuum. Therefore, we can consider this quantity being reducible from the vacuum expectation of appropriate scalar field ϕ by introducing the contribution to the lagrangian in the form of $\phi\Lambda_0^3$ that reproduces the cosmological constant at some $\phi \mapsto \langle\phi\rangle = \phi_0$. Without the gravity, the value of cosmological constant is irrelevant to the physics and it can take any value that corresponds to the global shift symmetry $\phi \mapsto \phi + \phi_c$, while the action can contain any scalar terms dependent on $\partial_\mu\phi$ and trivial flat potential of ϕ . These properties are characteristic for the inflaton field. This is gravity that is responsible for a generation of terms breaking the global shift symmetry, particularly, a non-flat potential as well as the kinetic term for ϕ that makes it to be the dynamical field of inflaton, we believe.

Finally, we can straightforwardly generalize the mechanism to the calculation of vacuum energy for the nonhomogeneous scalar field. In this case, the bare expression for the average tensor of energy and momentum

$$\langle \text{vac} | T_\mu^\nu | \text{vac} \rangle = \langle \text{vac} | \left\{ \partial_\mu \phi \partial^\nu \phi - \frac{1}{2} \delta_\mu^\nu (\partial\phi)^2 + \frac{1}{2} \delta_\mu^\nu m^2 \phi^2 \right\} | \text{vac} \rangle$$

can be written as the integral

$$\langle \text{vac} | T_\mu^\nu | \text{vac} \rangle = \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i0} \left\{ k_\mu k^\nu - \frac{1}{2} \delta_\mu^\nu (k^2 - m^2) \right\}.$$

After the Wick rotation $k_0 = ik_4$ to the Euclidean space, wherein $k^2 = -k_E^2$, we get

$$\langle \text{vac} | T_\mu^\nu | \text{vac} \rangle = \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^2 + m^2} \left\{ -k_{E\mu} k_E^\nu + \frac{1}{2} \delta_\mu^\nu (k_E^2 + m^2) \right\},$$

and the isotropic integration makes the replacement

$$k_{E\mu} k_E^\nu \mapsto \frac{1}{4} k_E^2 \delta_\mu^\nu,$$

resulting in the expected divergent expression with the vacuum signature of δ_μ^ν in the tensor structure,

$$\langle \text{vac} | T_\mu^\nu | \text{vac} \rangle = \delta_\mu^\nu \frac{1}{4} \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^2 + m^2} \{ k_E^2 + 2m^2 \}.$$

At this stage we can use the introduction of effective number of quanta with Euclidean four-momentum k_E , $n = n(k_E^2)$ in order to get the true expression for the energy density of suppressed vacuum fluctuations,

$$\rho_\Lambda = \frac{1}{4} \int \frac{d^4 k_E}{(2\pi)^4} \frac{e^{-n(k_E^2)}}{k_E^2 + m^2} \{ k_E^2 + 2m^2 \}. \quad (9)$$

This value is finite, if n increases with k_E^2 , say, polynomially. So, setting the ansatz of linear dependence

$$n(k_E^2) = \bar{n} + \frac{k_E^2}{\Lambda^2}$$

we find

$$\rho_\Lambda = \frac{1}{32\pi^2} e^{-\bar{n}} \left\{ \bar{\Lambda}^4 + m^2 \bar{\Lambda}^2 - m^4 \left(\ln \frac{\bar{\Lambda}^2}{m^2} - \gamma_E + \mathcal{O} \left(\frac{m^2}{\bar{\Lambda}^2} \ln \frac{\bar{\Lambda}^2}{m^2} \right) \right) \right\},$$

where $\gamma_E \approx 0.5772$ is the Euler gamma. In the limit of $\bar{n} \mapsto n$ given in the case of spatially global field, the cosmological constant gets the leading term of $\bar{\Lambda}^4$ by the virtual modes and subleading contribution of $m^2 \bar{\Lambda}^2$ analogous to the expression derived for the zero-point modes of global field⁴ at $\bar{\Lambda} \mapsto m_{\text{Pl}}$. Anyway, the suppression factor gets the form of exponentiating the effective number of quanta for the spatially global field. Thus, the quantum description of vacuum energy density in the non-stationary state shows the justified difference from the naive expectations on the cosmological constant formed by fluctuations of the zero-point modes, i.e. the bare cosmological constant.

Note, that the energy density in the model of growing $n(k_E^2)$ formally becomes infinite, unless we introduce an evident cut off:

$$V_R \int_0^{\Lambda_{\text{cut}}} \frac{d^3 k}{(2\pi)^3} n(k^2) \sqrt{k^2 + m^2} = E_{\text{tot}},$$

by setting a finite total energy E_{tot} in the reference volume. Nevertheless, our consideration remains valid in the case of $\Lambda_{\text{cut}} \gg \bar{\Lambda}$.

Evidently, the offered mechanism looks to be not straightforwardly effective in the case of any fermionic field, when the occupation number takes only two values: zero and unit. However, the *global* fermionic field is not relevant to the cosmological constant, of course. At this point, it is important to note that we assign the bare cosmological constant to the sum over *all* contributions of physical fields in the word including fermionic fields, and, say, the Higgs boson, quark-gluon condensates and so on. This is the gravity who is the reason why *the bare cosmological constant is transformed into the real dynamical scalar field* with the vacuum quantum numbers of charges, i.e. into the inflaton, which is non-stationary excited from the state of minimal energy in the finite volume. Then, by means of the quantum-mechanical craft the excitation produces the suppression of bare cosmological constant.

We address this problem on various sources of total cosmological constant in new paper [30], wherein we argue on the pseudo-Goldstone nature of inflaton with respect to the global shift of energy scale of vacuum energy density. In this mechanism, the primary cosmological constant induced by all of actual contributions is matched to the bare cosmological constant of inflaton. This matching lifts the mentioned problem on the copious ingredients of total cosmological constant, we hope.

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⁴ If $\bar{\Lambda} \mapsto \Lambda_{\text{inf}}$, then one can expect that $\bar{\Lambda}^4 \sim m^2 m_{\text{Pl}}^2$ yielding $m \sim \Lambda_{\text{inf}}^2 / m_{\text{Pl}} \sim 10^{13}$ GeV.

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